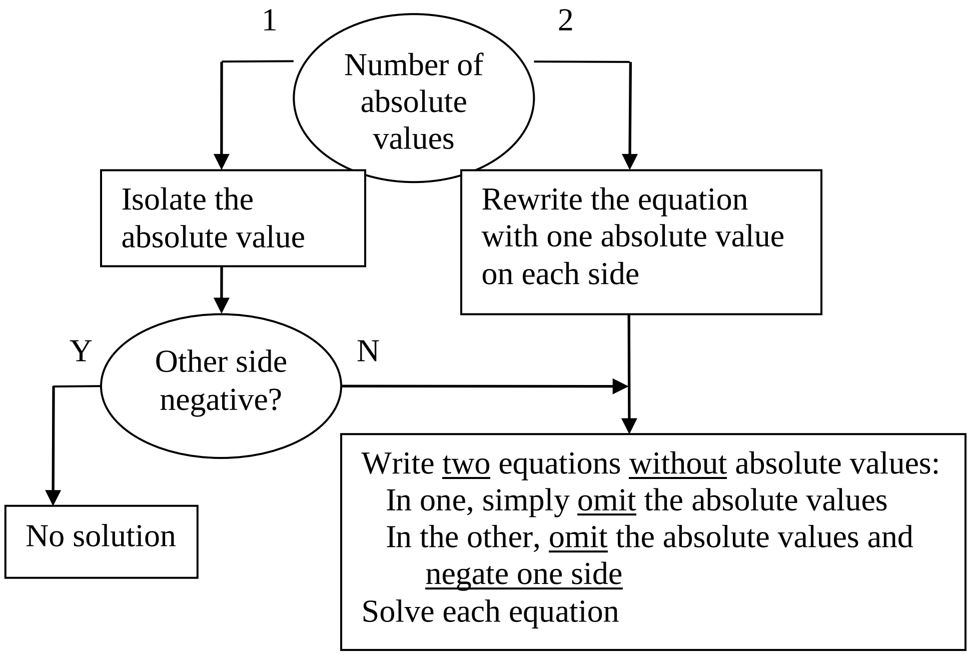
Success Center Math Tips

Solving Equations and Inequalities
in Intermediate Algebra

1. Absolute Value Equations



$$|3x-2|-3=1$$

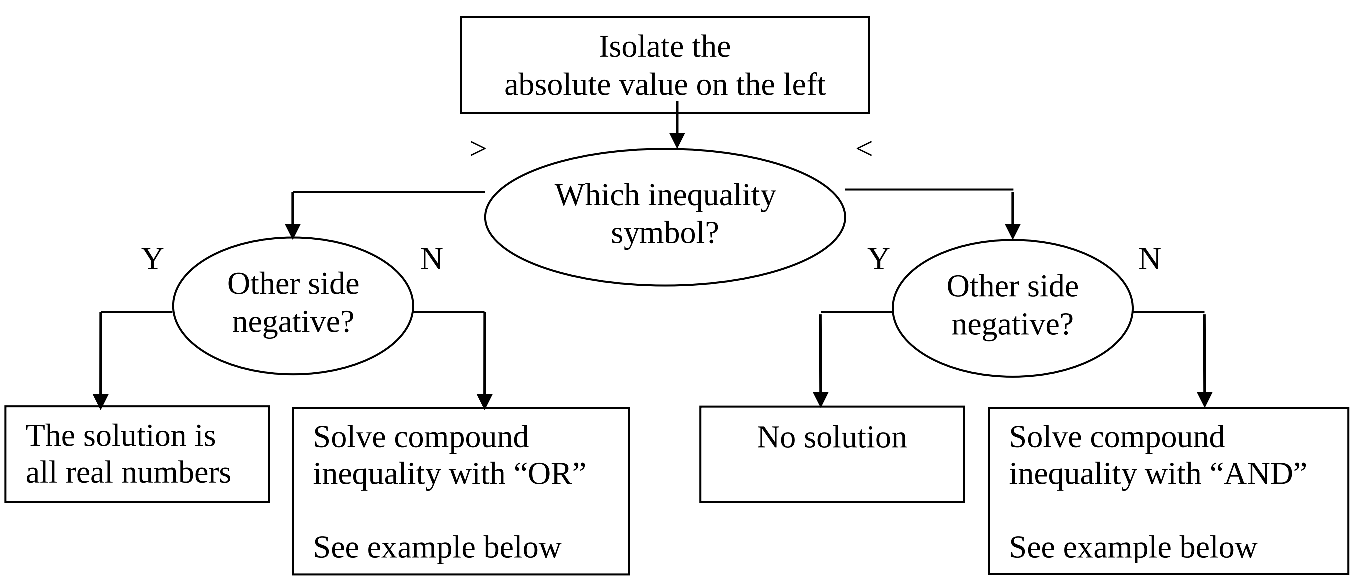
$$|3x-2|=4$$

$$3x-2=4 \quad \text{or} \quad 3x-2=-4$$

$$3x=6 \quad \text{or} \quad 3x=-2$$

$$x=2 \quad \text{or} \quad x=-\tfrac{2}{3}$$

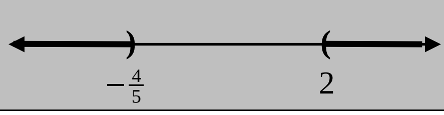
2. Absolute Value Inequalities



$$|5x-3|>7$$

$$5x-3>7 \quad \text{or} \quad 5x-3<-7$$

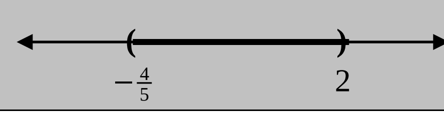
$$x>2 \quad \text{or} \quad x<-\tfrac{4}{5}$$



$$|5x-3|<7$$

$$-7<5x-3<7$$

$$-\tfrac{4}{5}<x<2$$



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3. Polynomial Equations

$$x^3 - 4x^2 = 12x$$

Move all terms to the same side of the equation and place them in descending order

$$x^3 - 4x^2 - 12x = 0$$

Factor the resulting polynomial

$$x(x^2 - 4x - 12) = 0$$
$$x(x-6)(x+2) = 0$$

Set each factor equal to zero and solve the resulting equations

$$x = 0 \quad \text{or} \quad x - 6 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 0 \quad \text{or} \quad x = 6 \quad \text{or} \quad x = -2$$

4. Fractional Equations

$$\frac{6}{x-3} - \frac{3}{8} = \frac{21}{4x-12}$$

$$\frac{6}{x-3} - \frac{3}{8} = \frac{21}{4(x-3)}$$

Any denominator with a variable?

Y

Write down all values which the variable cannot have

$$x \neq 3$$

N

Multiply both sides of the equation by the LCD to clear all fractions

$$8(x-3)\left(\frac{6}{x-3} - \frac{3}{8}\right) = \frac{21}{4(x-3)}(8)(x-3)$$

$$8(6) - 3(x-3) = 21(2)$$

$$48 - 3x + 9 = 42$$

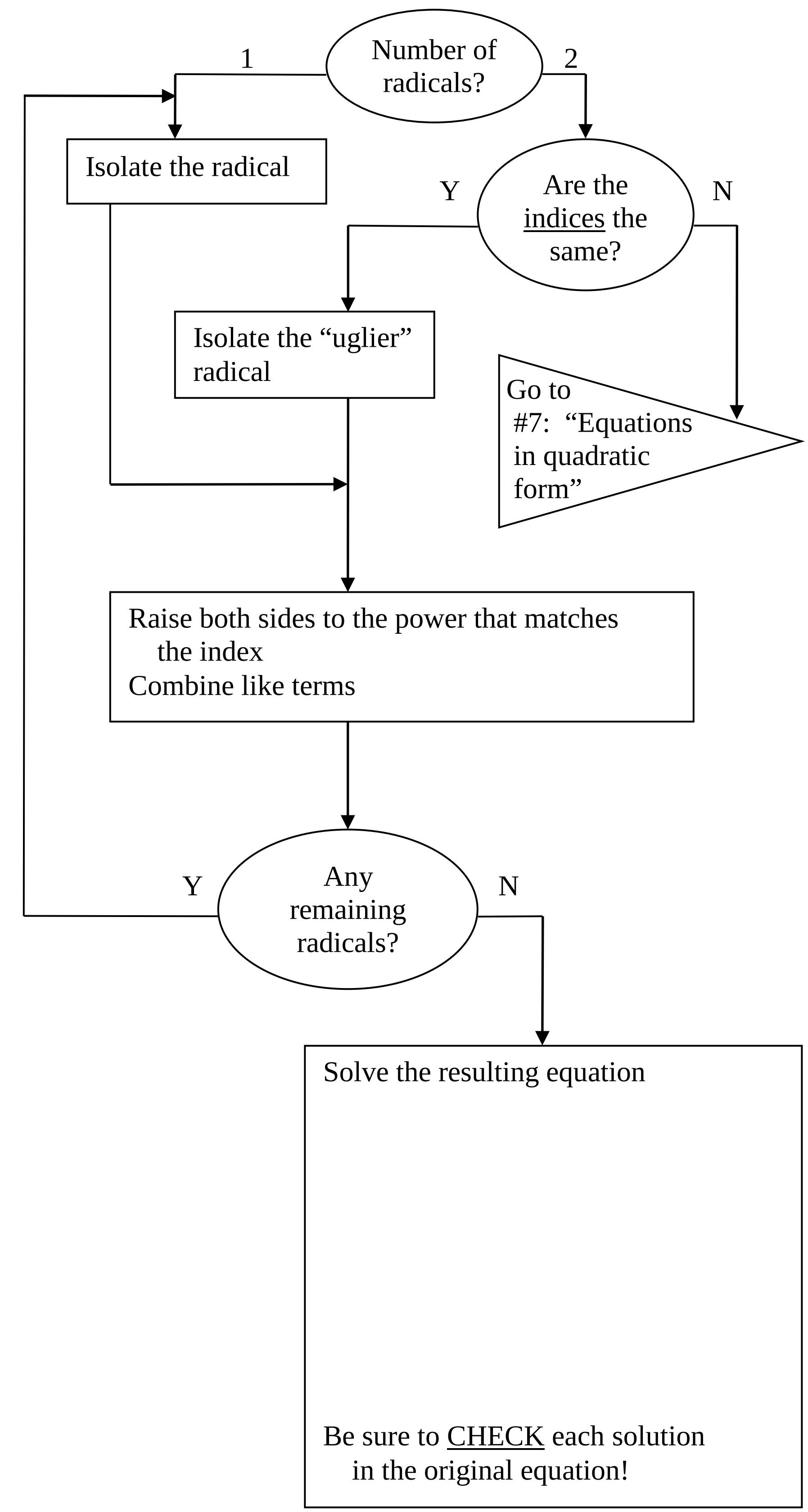
Solve the resulting equation—eliminate any values which the variable cannot have

$$57 - 3x = 42$$

$$-3x = -15$$

$$x = 5$$

5. Radical Equations

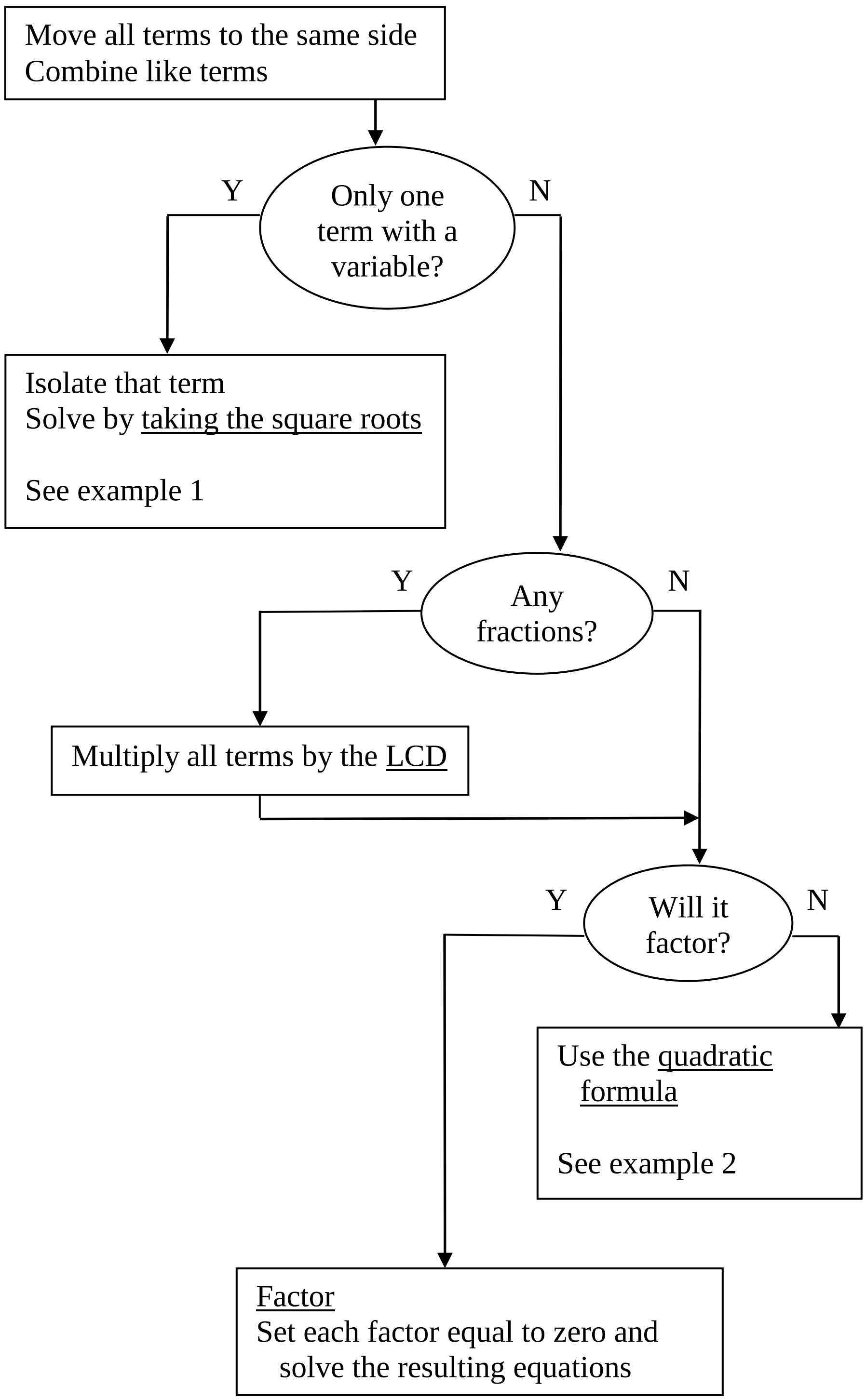


1.
$$\sqrt{2x+1}+8=15$$
$$\sqrt{2x+1}=7$$
$$(\sqrt{2x+1})^2=7^2$$
$$2x+1=49$$
$$x=24$$

2.
$$\sqrt{3x+4}+\sqrt{x}=2$$
$$\sqrt{3x+4}=2-\sqrt{x}$$
$$(\sqrt{3x+4})^2=(2-\sqrt{x})^2$$
$$3x+4=4-4\sqrt{x}+x$$
$$3x=-4\sqrt{x}+x$$
$$2x=-4\sqrt{x}$$
$$x=-2\sqrt{x}$$
$$(x)^2=(-2\sqrt{x})^2$$
$$x^2=4x$$
$$x^2-4x=0$$
$$x(x-4)=0$$
$$x=0 \quad \text{or} \quad x-4=0$$
$$x=0 \quad \text{or} \quad x=4$$

ONLY $x=0$ works in the original equation!

6. Quadratic Equations



1. $5x^2+2x-16=x^2+2x+20$

$$4x^2-36=0$$

$$4x^2=36$$

$$x^2=9$$

$$\sqrt{x^2}=\pm\sqrt{9}$$

$$x=\pm 3$$

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2. $x^2+3x+1=2x^2-5x+3$

$$x^2-8x+2=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=1$$
$$b=-8$$
$$c=2$$

$$x=\frac{8\pm\sqrt{64-4(1)(2)}}{2}$$

$$x=\frac{8\pm\sqrt{56}}{2}$$

$$x=\frac{8\pm 2\sqrt{14}}{2}$$

$$x=4\pm\sqrt{14}$$

7. Equations in Quadratic Form

These are equations that may be written in the form

$$a(\qquad)^2 + b(\qquad) + c = 0$$

where a, b, and c are numbers and where the parentheses may contain any algebraic expression.

Some examples are:

$$(3x+1)^2 + 5(3x+1) + 4 = 0$$

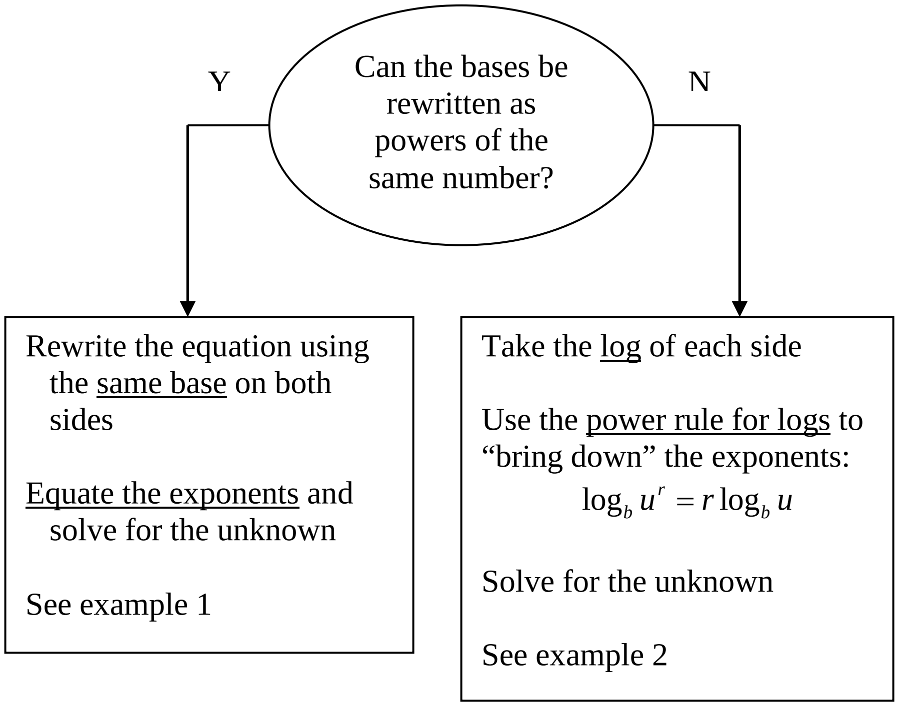
$$2x^4 - 3x^2 + 1 = 0 \quad \text{or} \quad 2(x^2)^2 - 3(x^2) + 1 = 0$$

$$6x^{-2} + x^{-1} - \frac{3}{2} = 0 \quad \text{or} \quad 6(x^{-1})^2 + (x^{-1}) - \frac{3}{2} = 0$$

$$\sqrt{x} - 4\sqrt[4]{x} + 5 = 0 \quad \text{or} \quad (\sqrt[4]{x})^2 - 4(\sqrt[4]{x}) + 5 = 0$$

	$(3x+1)^2 + 5(3x+1) + 4 = 0$
Give a “name” to the algebraic expression within parentheses, say u	Let $u = 3x+1$
Use this name to <u>rewrite</u> the equation as $au^2 + bu + c = 0$	Then $u^2 + 5u + 4 = 0$
Solve this quadratic equation to find u	$(u+4)(u+1) = 0$ $u+4=0$ or $u+1=0$ $u=-4$ or $u=-1$
For each value of u obtained, write an equation using the expression within parentheses from the original equation: u = ( )	Since $u = 3x+1$ $3x+1=-4$ or $3x+1=-1$
Solve each equation for the variable within the parentheses	$3x=-5$ or $3x=-2$ $x=-\frac{5}{3}$ or $x=-\frac{2}{3}$

8. Exponential Equations



1.

$$9^{x+2} = 27^x$$

$$(3^2)^{2x+2} = (3^3)^x$$

$$2x + 4 = 3x$$

$$x = 4$$

2.

$$6^{2x+1} = 5^{x+2}$$

$$\log 6^{2x+1} = \log 5^{x+2}$$

$$(2x+1)\log 6 = (x+2)\log 5$$

$$2x\log 6 + \log 6 = x\log 5 + 2\log 5$$

$$2x\log 6 - x\log 5 = 2\log 5 - \log 6$$

$$(2\log 6 - \log 5)x = 2\log 5 - \log 6$$

$$x = \frac{2\log 5 - \log 6}{2\log 6 - \log 5}$$

$$x \approx .7229$$

9. Logarithmic Equations

Move all terms <u>with a log</u> to one side of the equation and all terms <u>without a log</u> to the other side

Use the rules for logarithms to rewrite the side with all the logs as a <u>single log</u>:

$$\log_b uv = \log_b u + \log_b v$$

$$\log_b \frac{u}{v} = \log_b u - \log_b v$$

$$\log_b u^r = r\log_b u$$

Rewrite the resulting equation in <u>exponential form</u> and solve

$$\log x = 2 + \log(x-1)$$

$$\log x - \log(x-1) = 2$$

$$\log\frac{x}{x-1} = 2$$

$$10^2 = \frac{x}{x-1}$$

$$100 = \frac{x}{x-1}$$

$$100(x-1) = x$$

$$100x - 100 = x$$

$$99x = 100$$

$$x = \frac{100}{99}$$