1. Absolute Value Equations

- **Number of absolute values**
- **Isolate the absolute value**
- **Rewrite the equation with one absolute value on each side**
- **Other side negative?**
  - Y: Write two equations without absolute values:
    - In one, simply omit the absolute values
    - In the other, omit the absolute values and negate one side
    - Solve each equation
  - N: No solution

| 3x – 2 | –3 = 1
| 3x – 2 | = 4

3x – 2 = 4 or 3x – 2 = –4
3x = 6 or 3x = –2
x = 2 or x = –\(\frac{2}{3}\)

2. Absolute Value Inequalities

- **Isolate the absolute value on the left**
- **Which inequality symbol?**
  - >: The solution is all real numbers
  - <: Solve compound inequality with “OR”
    - See example below
- **Other side negative?**
  - Y: Solve compound inequality with “AND”
    - See example below
  - N: No solution

| 5x – 3 | > 7
\[ 5x – 3 > 7 \quad \text{or} \quad 5x – 3 < –7 \]
\[ x > 2 \quad \text{or} \quad x < –\frac{4}{5} \]

| 5x – 3 | < 7
\[ –7 < 5x – 3 < 7 \]
\[ –\frac{4}{5} < x < 2 \]
3. Polynomial Equations

Move all terms to the same side of the equation and place them in descending order

Factor the resulting polynomial

Set each factor equal to zero and solve the resulting equations

4. Fractional Equations

Any denominator with a variable?

Write down all values which the variable cannot have

Multiply both sides of the equation by the LCD to clear all fractions

Solve the resulting equation—eliminate any values which the variable cannot have

\[ x^3 - 4x^2 = 12x \]
\[ x^3 - 4x^2 - 12x = 0 \]
\[ x(x^2 - 4x - 12) = 0 \]
\[ x(x - 6)(x + 2) = 0 \]
\[ x = 0 \text{ or } x - 6 = 0 \text{ or } x + 2 = 0 \]
\[ x = 0 \text{ or } x = 6 \text{ or } x = -2 \]

\[ \frac{6}{x - 3} - \frac{3}{8} = \frac{21}{4x - 12} \]
\[ \frac{6}{x - 3} - \frac{3}{8} = \frac{21}{4(x - 3)} \]
\[ x \neq 3 \]

\[ 8(x - 3)(\frac{6}{x - 3} - \frac{3}{8}) = \frac{21}{4(x - 3)} (8)(x - 3) \]
\[ 8(6) - 3(x - 3) = 21(2) \]
\[ 48 - 3x + 9 = 42 \]
\[ 57 - 3x = 42 \]
\[ -3x = -15 \]
\[ x = 5 \]
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Solving Equations and Inequalities in Intermediate Algebra

5. Radical Equations

1. \( \sqrt{2x+1} + 8 = 15 \)
   \( \sqrt{2x+1} = 7 \)
   \( (\sqrt{2x+1})^2 = 7^2 \)
   \( 2x + 1 = 49 \)
   \( x = 24 \)

2. \( \sqrt{3x + 4} + \sqrt{x} = 2 \)
   \( \sqrt{3x + 4} = 2 - \sqrt{x} \)
   \( (\sqrt{3x + 4})^2 = (2 - \sqrt{x})^2 \)
   \( 3x + 4 = 4 - 4\sqrt{x} + x \)
   \( 3x = -4\sqrt{x} + x \)
   \( 2x = -4\sqrt{x} \)
   \( x = -2\sqrt{x} \)
   \( (x)^2 = (-2\sqrt{x})^2 \)
   \( x^2 = 4x \)
   \( x^2 - 4x = 0 \)
   \( x(x - 4) = 0 \)
   \( x = 0 \) or \( x - 4 = 0 \)
   \( x = 0 \) or \( x = 4 \)

ONLY \( x = 0 \) works in the original equation!
6. Quadratic Equations

1. \(5x^2 + 2x - 16 = x^2 + 2x + 20\)
   \[4x^2 - 36 = 0\]
   \[4x^2 = 36\]
   \[x^2 = 9\]
   \[\sqrt{x^2} = \pm\sqrt{9}\]
   \[x = \pm 3\]

2. \(x^2 + 3x + 1 = 2x^2 - 5x + 3\)
   \[x^2 - 8x + 2 = 0\]
   \[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
   \[a = 1\]
   \[b = -8\]
   \[c = 2\]
   \[x = \frac{8 \pm \sqrt{64 - 4(1)(2)}}{2}\]
   \[x = \frac{8 \pm \sqrt{56}}{2}\]
   \[x = \frac{8 \pm 2\sqrt{14}}{2}\]
   \[x = 4 \pm \sqrt{14}\]
7. Equations in Quadratic Form

These are equations that may be written in the form

\[ a(\ )^2 + b(\ ) + c = 0 \]

where \( a, b, \) and \( c \) are numbers and where the parentheses may contain any algebraic expression.

Some examples are:

\[ (3x + 1)^2 + 5(3x + 1) + 4 = 0 \]
\[ 2x^4 - 3x^2 + 1 = 0 \quad \text{or} \quad 2(x^2)^2 - 3(x^2) + 1 = 0 \]
\[ 6x^{-2} + x^{-1} - \frac{3}{2} = 0 \quad \text{or} \quad 6(x^{-1})^2 + (x^{-1}) - \frac{3}{2} = 0 \]
\[ \sqrt{x} - 4\sqrt{x} + 5 = 0 \quad \text{or} \quad (\sqrt{x})^2 - 4(\sqrt{x}) + 5 = 0 \]

Give a “name” to the algebraic expression within parentheses, say \( u \)

Use this name to rewrite the equation as \( au^2 + bu + c = 0 \)

Solve this quadratic equation to find \( u \)

For each value of \( u \) obtained, write an equation using the expression within parentheses from the original equation:

\[ u = (\ ) \]

Solve each equation for the variable within the parentheses

\[ (3x + 1)^2 + 5(3x + 1) + 4 = 0 \]
Let \( u = 3x + 1 \)

Then \( u^2 + 5u + 4 = 0 \)

\[ (u + 4)(u + 1) = 0 \]
\( u + 4 = 0 \quad \text{or} \quad u + 1 = 0 \)
\( u = -4 \quad \text{or} \quad u = -1 \)

Since \( u = 3x + 1 \)
\( 3x + 1 = -4 \quad \text{or} \quad 3x + 1 = -1 \)
\( 3x = -5 \quad \text{or} \quad 3x = -2 \)
\( x = -\frac{5}{3} \quad \text{or} \quad x = -\frac{2}{3} \)
8. Exponential Equations

Can the bases be rewritten as powers of the same number?

Y

Rewrite the equation using the same base on both sides

N

Equate the exponents and solve for the unknown

See example 1

Take the log of each side

Use the power rule for logs to “bring down” the exponents:

\[ \log_b u^r = r \log_b u \]

Solve for the unknown

See example 2

1. \[ 9^{x+2} = 27^x \]
   \[ (3^2)^{2x+2} = (3^3)^x \]
   \[ 2x + 4 = 3x \]
   \[ x = 4 \]

2. \[ 6^{2x+1} = 5^{x+2} \]
   \[ \log 6^{2x+1} = \log 5^{x+2} \]
   \[ (2x + 1) \log 6 = (x + 2) \log 5 \]
   \[ 2x \log 6 + \log 6 = x \log 5 + 2 \log 5 \]
   \[ 2x \log 6 - x \log 5 = 2 \log 5 - \log 6 \]
   \[ (2 \log 6 - \log 5)x = 2 \log 5 - \log 6 \]
   \[ x = \frac{2 \log 5 - \log 6}{2 \log 6 - \log 5} \]
   \[ x \approx 0.7229 \]

9. Logarithmic Equations

Move all terms with a log to one side of the equation and all terms without a log to the other side

Use the rules for logarithms to rewrite the side with all the logs as a single log:

\[ \log_b uv = \log_b u + \log_b v \]
\[ \log_b \frac{u}{v} = \log_b u - \log_b v \]
\[ \log_b u^r = r \log_b u \]

Rewrite the resulting equation in exponential form and solve

\[ \log x = 2 + \log(x - 1) \]
\[ \log x - \log(x - 1) = 2 \]
\[ \log \frac{x}{x - 1} = 2 \]
\[ 10^2 = \frac{x}{x - 1} \]
\[ 100 = \frac{x}{x - 1} \]
\[ 100(x - 1) = x \]
\[ 100x - 100 = x \]
\[ 99x = 100 \]
\[ x = \frac{100}{99} \]